Schopohl and Dolgov reply: let us first summarize the pertinent result of the subject letter[1], before addressing statements made in the preceding interesting and stimulating comments[2],[3]. We[1] consider the *total* electromagnetic contribution to the free energy $\mathcal{F}$ for a *fixed* (externally controlled) current distribution $\mathbf{j}_{ext}(\mathbf{q})$ in a superconductor. To ease comparison of our conclusions[1] with the foundational comment of Volovik[2] we rewrite $\mathcal{F}$ in the equivalent form[1]

$$\mathcal{F} = -\frac{2\pi}{c^2}\int \frac{d^3q}{(2\pi)^3}\frac{|\mathbf{j}_{ext}(\mathbf{q})|^2}{q^2+\frac{1}{\lambda^2(\mathbf{q},T)}} \tag{1}$$

where $\lambda(\mathbf{q},T)$ is the temperature dependent magnetic penetration depth (**an eigenvalue of the electromagnetic kernel**). From this one readily calculates the electromagnetic part of the entropy $S(T) = -\frac{\partial \mathcal{F}}{\partial T}$ for any externally controlled (*finite*) current distribution $\mathbf{j}_{ext}(\mathbf{q})$:

$$S(T) = -\frac{2\pi}{c^2}\int \frac{d^3q}{(2\pi)^3}\frac{\partial}{\partial T}\left[\frac{1}{\lambda^2(\mathbf{q},T)}\right]\cdot\frac{|\mathbf{j}_{ext}(\mathbf{q})|^2}{\left[q^2+\frac{1}{\lambda^2(\mathbf{q},T)}\right]^2} \tag{2}$$

We argued[1], that a strictly linear T-dependence of the magnetic penetration depth (MPD) in a superconductor, of the form $\lambda(T)-\lambda(0)\propto T$, violates indeed the third law of thermodynamics.

Volovik[2] discusses a situation, where a stationary flow (associated with an *uncharged* superfluid) circulates in a ring. He also suggests[2] to compare a charged system with an uncharged one: the externally controlled current density $\mathbf{j}_{ext}$ should be related to the superfluid velocity $\mathbf{v}_s$, the magnetic penetration depth $\lambda(T)$ should be identified with the superfluid density $\rho_s(T)=\rho-\rho_n(T)$, the free energy $\mathcal{F}$ in Eq.(1) should be equivalent to the kinetic energy $\frac{1}{2}\rho_s(T)\,\mathbf{v}_s^2$ of the flow of a neutral superfluid[2]. However, such an identification is deceptively simple:

a) $\mathbf{j}_{ext}$ is externally controlled, and henceforth strictly independent on T, while $\mathbf{v}_s$, as discussed by Volovik, may in principle display a T-dependence. In some cases the T-dependence of $\mathbf{v}_s$ may be irrelevant, in other cases it may be important, depending on circumstances how $\mathbf{v}_s$ is measured. For example, what is the meaning of $\mathbf{v}_s$ in a microwave absorption experiment?

b) $\mathcal{F}$ describes a lot more than just the kinetic energy of the superflow, since $\mathcal{F}$ explicitely takes into account the full electromagnetic interaction of the ordered medium (in our case the superconductor) with the externally controlled c-number source $\mathbf{j}_{ext}$ (see Ref.[5] for an authoritative discussion of the electrodynamics of ordered macroscopic media). Also, Volovik restricts his discussion to the hydrodynamic limit $|\mathbf{q}|\to 0$, while $\mathcal{F}$ takes into account also the non local effects, important for wavevectors around $|\mathbf{q}|\cdot\lambda(T)\gtrsim 1$.

c) Volovik states[2] that the total current density $\mathbf{j}$ should be proportional to $\mathbf{v}_s$, via $\mathbf{j}=\rho_s\mathbf{v}_s$, so that the 'response' ρ_s is defined in the limit $|\mathbf{v}_s|\to 0$, while the Nernst principle requires first $T\to 0$ at finite $\mathbf{v}_s$. Should we distrust electromagnetic linear response theory? Can it not be applied to a charged superfluid at low temperatures when there exists a finite superflow in the ground state? Our answer is that the transversal electromagnetic linear response function[5], defined via[1]

$$\left[\mathbf{q}^2-\frac{\omega^2}{c^2}\varepsilon_{tr}(\mathbf{q},\omega)\right]\mathbf{A}(\mathbf{q},\omega) = \frac{4\pi}{c}\mathbf{j}_{ext}(\mathbf{q},\omega) \tag{3}$$

is physically correct for any externally controlled c-number current $\mathbf{j}_{ext}$, provided $|\mathbf{j}_{ext}|$ is *small* compared to the microscopic atomic (molecular) currents in the matter. Indeed, the physical notion and subsequent distinction between (weak) *external* fields and (possibly strong) *local* fields inside the matter adequately addresses this point. Only under extreme conditions external fields become, eventually, comparable to the strength of local fields, for example in laser physics. Within the range of validity of linear optics, however, the linear response of a superconductor, represented by the *transversal* dielectric function ε_{tr}, should be physically adequate to describe the reaction of a superconductor to any externally controlled electromagnetic influence $\mathbf{j}_{ext}$, and this at all temperatures T.

This view is also taken by Hirschfeld et al.[3]. These authors discuss the static $\mathbf{q}$-dependent electromagnetic response kernel $\lim_{\omega\to 0}\frac{\omega^2}{c^2}\left[1-\mathrm{Re}\,\varepsilon_{tr}(\mathbf{q},\omega)\right]$, which they construct (using standard linear response theory) out of a product of two Matsubara Green's functions having as its source the weak coupling BCS theory. Their electromagnetic kernel applies for conventional as well as unconventional pairing symmetry. The interesting point made by Hirschfeld et al. is, that irrespective of the orientation of $\mathbf{q}$ with respect to the ab-plane of a layered superconductor, there exists a tiny correction of order q^2, that should stabilize a pure $d_{x^2-y^2}$ symmetry BCS pairing groundstate at low temperatures, even in the exceptional case[1] when $\mathbf{q}\cdot\mathbf{v}_F=0$. They estimate the crossover temperature T^*, below which a deviation of the linear $T-$dependence of MPD should be detected, around $k_BT^*\simeq\frac{\hbar^2\omega_{pl}^2}{2mc^2}$. Such a crossover temperature T^* might be outside the predictive power of the (non relativistic) electromagnetic response theory, though. But accepting their argument, the electromagnetic kernel becomes intrinsically non local. This leads trivially to a non linear T-dependence of MPD fulfilling $\lim_{T\to 0}\frac{\partial\lambda(\mathbf{q},T)}{\partial T}=0$, in accordance with the Nernst principle[1]. We agree: a $d_{x^2-y^2}$-pairing groundstate may be stable at low temperatures, but it will *not* display a linear $T-$dependence of MPD due to non local effects[4],[3]. For $T\to 0$ the conceptual distinction between a *non local* electromagnetic response kernel on one hand and a *local* superfluid density ρ_s on the other hand becomes important. The answer to the question posed in the title of the subject letter[1] is no.

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